Computational Seismology: An Introduction

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Pathways of Seismology Study

Observation

source

Earth

instrument

Modeling

instrument response
2008 Wenchuan, China, Earthquake

Before earthquake

After earthquake
Rupture of Wenchuan Earthquake

Rupture duration: ~100 s
Rupture length: ~300 km

Wang et al. (2008)
March 11, 2011, M9.0 Japan Earthquake

Rupture duration: ~180 s
Rupture length: ~500 km

Hayes, USGS (2011)
Major Factors Influencing Ground Motion

Amplification by soft near-surface material

Focusing of energy by material boundaries

Earthquake source rupture directivity
Surface Waves Cause Most Damages

Snapshots of Denali, Alaska, Earthquake

Surface waves (slowest, strongest)
S waves (slower, stronger)
P wave (fast, weak)

Komatitsch et al. (2005)
Amplification by Soft Sediments

Soft sedimentary basin can amplify ground motion
1994 Northridge (CA) Earthquake

Basin geometry focusing can further amplify ground motion.
Directivity Effect on Ground Motion

Visualization: Amit Chourasia
Steve Cutchin

Rendering: Marcio Faerman

Compositing: Amit Chourasia
Jon C. Meyer

Visualization Services
San Diego Supercomputer Center
University of California San Diego
http://visservices.npaci.edu/

Click to show movie
Reliable Ground Motion Prediction Requires

Earth structure model

Reliable model of the source

Computational tools
Observation
Deploy instruments to collect *waveform records*
Waveforms carry information about the *sources* of earthquake and the *structure* along the paths between source and station

Prediction
Calculate *waveforms* for models of *source* and *structure*
Issue: *efficiency* and *accuracy*

Measurement
Compare *predicted* and *recorded* waveforms
Differences (residuals, anomalies) serve as *data* to refine *source* and *structure* models.

Set Up Inverse Problem
Relate data and model perturbations
Issue: (1) *linear relation* (2) both *source* and *structure* models

Model Update

Inversion
Solve the inverse problem
Collection of Global Earthquake Data

2007/02/07 18:54:59 (N=844, Mw=5.5)
Observations vs. Predictions

Yang, Zhao & Hung (2010)
A major task of computational seismology is to solve the wave equation.

\[ \rho \ddot{u} = \nabla \cdot \tau + f \]

Hooke’s Law: \( \tau = C : \varepsilon = C : \left\{ \frac{1}{2} \left[ (\nabla u) + (\nabla u)^T \right] \right\} = C : (\nabla u) \)

Wave Equation:

Simplified Earth model: 1D, simple geometry
- Semi-analytic method: normal modes
- High-frequency approximation: ray theory (optics)

Realistic Earth model: 3D, irregular geometry, topography
- finite-difference method (FDM)
- finite-element method -- spectral-element method (SEM)

A major task of computational seismology is to solve the wave equation.
Finite-Difference Method (FDM)

\[ \rho \ddot{\mathbf{u}} = \nabla \cdot \mathbf{\tau} + \mathbf{f} \quad \mathbf{\tau} = C : \varepsilon = C : \left\{ \frac{1}{2} \left[ (\nabla \mathbf{u}) + (\nabla \mathbf{u})^T \right] \right\} = C : (\nabla \mathbf{u}) \]

(1) Discretize the medium and time by a (usually uniform) spatial-temporal grid.

(2) Approximate the differential equation by finite difference schemes.
Simple Finite-Difference Schemes

From the Taylor expansion for function $\Phi(x)$:

$$
\Phi(x + h) = \Phi(x) + h \partial_x \Phi(x) + \frac{1}{2} h^2 \partial_x^2 \Phi(x) + \frac{1}{6} h^3 \partial_x^3 \Phi(x) + O(h^4)
$$

$$
\Phi(x - h) = \Phi(x) - h \partial_x \Phi(x) + \frac{1}{2} h^2 \partial_x^2 \Phi(x) - \frac{1}{6} h^3 \partial_x^3 \Phi(x) + O(h^4)
$$

Forward- and backward difference formula for 1\textsuperscript{st}-order derivative:

$$
\partial_x \Phi(x) = \frac{1}{h} [\Phi(x + h) - \Phi(x)] + O(h^2)
$$

1\textsuperscript{st}-order accuracy

$$
\partial_x \Phi(x) = \frac{1}{h} [\Phi(x) - \Phi(x - h)] + O(h^2)
$$

Central-difference formula for 1\textsuperscript{st}-order derivative:

$$
\partial_x \Phi(x) = \frac{1}{2h} [\Phi(x + h) - \Phi(x - h)] + O(h^3)
$$

2\textsuperscript{nd}-order accuracy

Central-difference formular for 2\textsuperscript{nd}-order derivative:

$$
\partial_x^2 \Phi(x) = \frac{1}{h^2} [\Phi(x + h) - 2\Phi(x) + \Phi(x - h)] + O(h^4)
$$

3\textsuperscript{rd}-order accuracy
Higher-Order Finite-Difference Schemes

From the Taylor expansions:

\[
\Phi(x + h) = \Phi(x) + h\partial_x \Phi(x) + \frac{1}{2} h^2 \partial_x^2 \Phi(x) + \frac{1}{6} h^3 \partial_x^3 \Phi(x) + \frac{1}{24} h^4 \partial_x^4 \Phi(x) + O(h^5)
\]

\[
\Phi(x - h) = \Phi(x) - h\partial_x \Phi(x) + \frac{1}{2} h^2 \partial_x^2 \Phi(x) - \frac{1}{6} h^3 \partial_x^3 \Phi(x) + \frac{1}{24} h^4 \partial_x^4 \Phi(x) - O(h^5)
\]

\[
\Phi(x + 3h) = \Phi(x) + 3h\partial_x \Phi(x) + \frac{9}{2} h^2 \partial_x^2 \Phi(x) + \frac{27}{6} h^3 \partial_x^3 \Phi(x) + \frac{64}{24} h^4 \partial_x^4 \Phi(x) + O(h^5)
\]

\[
\Phi(x - 3h) = \Phi(x) - 3h\partial_x \Phi(x) + \frac{9}{2} h^2 \partial_x^2 \Phi(x) - \frac{27}{6} h^3 \partial_x^3 \Phi(x) + \frac{64}{24} h^4 \partial_x^4 \Phi(x) - O(h^5)
\]

We obtain these relations:

\[
\Phi(x + h) - \Phi(x - h) = 2h\partial_x \Phi(x) + \frac{1}{3} h^3 \partial_x^3 \Phi(x) + O(h^5)
\]

\[
\Phi(x + 3h) - \Phi(x - 3h) = 6h\partial_x \Phi(x) + 9h^3 \partial_x^3 \Phi(x) + O(h^5)
\]

\[
27[\Phi(x + h) - \Phi(x - h)] - [\Phi(x + 3h) - \Phi(x - 3h)] = 48h\partial_x \Phi(x) + O(h^5)
\]

and the fourth-order finite-difference scheme:

\[
\partial_x \Phi(x) = \frac{9}{16h} [\Phi(x + h) - \Phi(x - h)] - \frac{1}{48h} [\Phi(x + 3h) - \Phi(x - 3h)] + O(h^5)
\]
Finite-Difference Implementation: 1D Case

1D wave equations (2\textsuperscript{nd} order):

\[ \rho \, \partial^2_t u = \partial_x (\tau), \quad \tau = k \partial_x u \]

Introducing velocity: \( v = \partial_t u \)

and we have two 1\textsuperscript{st}-order equations:

\[ \rho \, \partial_t v = \partial_x \tau, \quad \partial_t \tau = k \partial_x v \]

Applying the central-difference scheme: \( \partial_x \Phi(x) = \frac{1}{2h} [\Phi(x + h) - \Phi(x - h)] + O(h^3) \)

\[ \rho_i \frac{v_i^{n+1} - v_i^{n-1}}{2\Delta t} = \frac{\tau_i^{n+1} - \tau_i^{n-1}}{2\Delta x}, \quad \frac{\tau_i^{n+1} - \tau_i^{n-1}}{2\Delta t} = k_i \frac{v_{i+1}^n - v_{i-1}^n}{2\Delta x} \]

We get the 2\textsuperscript{nd}-order finite-difference equations:

\[ v_i^{n+1} = v_i^{n-1} + \frac{1}{\rho_i} \frac{\Delta t}{\Delta x} (\tau_i^{n+1} - \tau_i^{n-1}) \]

\[ \tau_i^{n+1} = \tau_i^{n-1} + k_i \frac{\Delta t}{\Delta x} (v_{i+1}^n - v_{i-1}^n) \]
Parallelization (128 Processes)

\[ v_{i}^{n+1} = v_{i}^{n-1} + \frac{1}{\rho_{i}} \frac{\Delta t}{\Delta x} (\tau_{i+1}^{n} - \tau_{i-1}^{n}) \]

\[ \tau_{i}^{n+1} = \tau_{i}^{n-1} + k_{i} \frac{\Delta t}{\Delta x} (v_{i+1}^{n} - v_{i-1}^{n}) \]
Finite-Difference Method (FDM)

\[ \rho \ddot{u} = \nabla \cdot \tau + f \]
\[ \tau = C : \varepsilon = C : \left\{ \frac{1}{2}[(\nabla u) + (\nabla u)^T] \right\} = C : (\nabla u) \]

(1) Discretize the medium and time by a (usually uniform) spatial-temporal grid.

(2) Approximate the differential equation by finite difference schemes. Only neighboring points are linked. Easy implementation of domain-decomposition for distributed parallel programs.

(3) FD grid is usually uniform for easy implementation and bookkeeping (advantage), but can not handle irregular geometry (shortcoming). Recent advances enable irregular grid.
**Spectral-Element Method (SEM)**

\[
\begin{align*}
\rho \dddot{\mathbf{u}} &= \nabla \cdot \mathbf{\tau} + f \\
\int_{\Omega} \rho \dddot{\mathbf{u}} \, d^3 r &= \int_{\Omega} \nabla \cdot \mathbf{\tau} \, d^3 r + \int_{\Omega} f \, d^3 r \\
\mathbf{\tau} &= \mathbf{C} : \mathbf{\varepsilon} = \mathbf{C} : \left\{ \frac{1}{2} [ (\nabla \mathbf{u}) + (\nabla \mathbf{u})^\top ] \right\} = \mathbf{C} : (\nabla \mathbf{u})
\end{align*}
\]

(1) Discretization of the medium by a volumetric mesh: a collection of (irregular) hexahedral cells (elements).

(2) Transformation from hexahedral cells to unit cubes.

(3) Integrate the wave equation in space to obtain its weak form.

(4) Expand all functions (displacement, stress, model, etc.) in spectral basis functions in each cell.

\[
u_k(\mathbf{x}) = \sum_{i=1}^{N} c_k^i B(\mathbf{x} - \mathbf{x}_k^i), \quad k = 1, 2, \ldots, M \quad M \text{ cells (elements) with } N \text{ control points in each cell}
\]

\[
\mathbf{G} \cdot \mathbf{c} = \mathbf{F}
\]

(5) SEM uses higher-order basis function; and the coefficient matrix \( \mathbf{G} \) is diagonal. Easy for inversion and parallelization.
Examples of Wave Propagation Simulations
Caltech’s Southern California ShakeMovie Portal

Welcome to ShakeMovie: Caltech’s Near Real Time Simulation of Southern California Seismic Events Portal. This portal has been designed to present the public with near real-time visualizations of recent significant seismic events in the Southern California Region. These movies are the results of simulations carried out on a large computer cluster. Movies are simulated based upon the software package SPECFEM3D. Earthquake movies will be available for download approximately 45 mins after the occurrence of an earthquake of magnitude 3.5 or greater.

Facts

When an earthquake occurs, seismic waves are generated which propagate away from the fault rupture.

Here we see the up-and-down velocity of the Earth’s surface. Strong blue waves indicate the surface is moving rapidly downward. Strong red waves indicate rapid upward motion.

When the waves pass through solid soil (sediments) they slow down and amplify. Waves speed up when they pass through hard rock.

The color of the waves oscillates between red and blue indicating alternating up and down motion.

shakemovie.caltech.edu
E-mail Dissemination of SoCal ShakeMovie

EMAIL NOTIFICATION
shakemovie.caltech.edu
Event Id: 10832573

There is new media now available for download on the recent seismic event:

Magnitude: 4.1
9 miles N of Ocotillo, CA
UTC: Thu Nov 4 19:39:59 2010
Latitude: 32.87
Longitude: -116.02

All movies for this event are available from:
http://shakemovie.caltech.edu/event?evid=10832573

The movies available include:

SOUTHERN CALIFORNIA
http://shakemovie.caltech.edu/dl?evid=10832573&product=socal&style=orange&size=small

LOS ANGELES BASIN
http://shakemovie.caltech.edu/dl?evid=10832573&product=la&style=orange&size=small

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PGV Map and Waveform Fitting
Princeton’s Global ShakeMovie Portal

Global ShakeMovie is Princeton University's Near Real Time Global Seismicity Portal (Tromp et al. 2010). It has been designed to present the public with near real time visualizations of recent earthquakes. These movies are the results of simulations carried out on a large computer cluster. Movies and 3D synthetic seismograms are calculated based upon the software package SPECFEM3D_GLOBE. Earthquake movies will be available for download approximately 1.5 hours (depending on magnitude) after the occurrence of a quake of magnitude 5.5 or greater.

**FACTS**

When an earthquake occurs, seismic waves are generated which propagate away from the fault rupture.

- Here we see the up-and-down velocity of the Earth’s surface. Strong blue waves indicate the surface is moving rapidly downward. Strong red waves indicate rapid upward motion.
- When the waves pass through soft soils (sediments) they slow down and amplify. Waves speed up when they pass through hard rock.
- The color of the waves oscillates between red and blue indicating alternating up and down motion.

Learn more...
Global ShakeMovie

Click to show movie

Tromp et al. (2011)
Shake Movie: Chi-Chi (Taiwan) Earthquake

Near-surface velocity

Click to show movie
Thank you!